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# A TWO-FREQUENCY METHOD FOR IMPEDANCE MEASUREMENT IN MICROWAVE FREQUENCY-HALVING NETWORKS

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ABSTRACT

The method described removes a major difficulty in the design of wideband frequency-halvers: the accurate characterization of the "pumped" input and output impedances at the plane of the varactors. The procedure permits the use of realistic input and output power levels with the input at twice the output frequency.

## INTRODUCTION

The optimum design of wideband frequency halvers requires that the "pumped" input and output varactor impedances $Z_{in}$ and $Z_{out}$ be known a priori. Since the halver is a threshold device [1,2] these impedances exist only for appropriate values of input power $P_{in}$ and input frequency $2\omega$. Therefore $Z_{in}(2\omega)$ and $Z_{out}(\omega)$ can only be measured in a functioning halver circuit. This dilemma can be resolved thus:

(a) Assuming the validity of the "pumped" varactor equations of Penfield and Rafuse [3], determine estimates $\tilde{Z}_{in}(2\omega)$ and $\tilde{Z}_{out}(\omega)$.
(b) Use $\tilde{Z}_{in}$ and $\tilde{Z}_{out}$ to design a prototype halver, e.g. with the aid of COMPACT [4].
(c) Measure the external impedances $Z_{in}^{ext}$ and $Z_{out}^{ext}$ of the resulting two-port.
(d) Knowing the halver network, "de-embed" the varactor impedances $Z_{in}$ and $Z_{out}$.
(e) Compare $Z_{in}$ and $Z_{out}$ with $\tilde{Z}_{in}$ and $\tilde{Z}_{out}$.
(f) Iterate as necessary.

Impedance measurement by conventional load-pull techniques requires determination of large-signal S-parameters. At a specific input frequency $2\omega$ and power $P_{in}$, the halver can be characterized by

$$\begin{bmatrix} b_1(2\omega) \\ b_2(\omega) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1(2\omega) \\ a_2(\omega) \end{bmatrix}, \qquad (1)$$

see Fig. 1. For a balanced halver, the port signals have minimum harmonic content [2]. From (1):

$$S_{11} = \left.\frac{b_1(2\omega)}{a_1(2\omega)}\right|_{a_2(\omega)=0}, \quad S_{12} = \left.\frac{b_1(2\omega)}{a_2(\omega)}\right|_{a_1(2\omega)=0}$$
$$S_{21} = \left.\frac{b_2(\omega)}{a_1(2\omega)}\right|_{a_2(\omega)=0}, \quad S_{22} = \left.\frac{b_2(\omega)}{a_2(\omega)}\right|_{a_1(2\omega)=0}. \qquad (2)$$

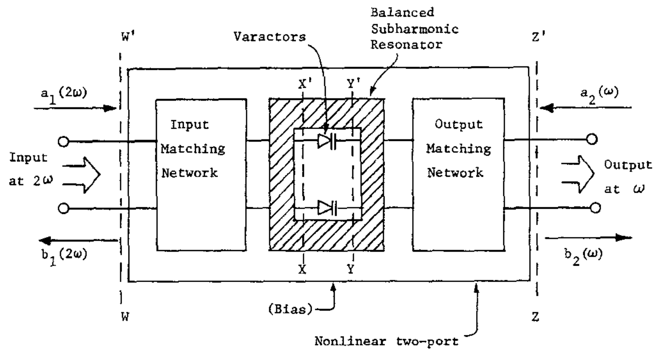


Fig. 1 Definition of 2-frequency S-matrix and reference planes.

A network analyser (NA) can be used to measure $S_{11}$ at $2\omega$ with the output terminated, but not the trans-frequency parameters $S_{12}$ and $S_{21}$ (which are of minor concern here). Determination of $S_{22}$ according to (2) is impossible because a reverse signal $a_2(\omega)$ has to be applied to the output port and the resulting reflected signal $b_2(\omega)$ measured, the input being terminated ($a_1$=0). However, this removes the drive power, resulting in cessation of halving.

## TWO-FREQUENCY MEASUREMENT TECHNIQUE

A solution employs a two-frequency variant of Mazumder's synthetic load technique[5]. From (1):

$$b_1(2\omega)/a_1(2\omega) = S_{11} + S_{12}\{a_2(\omega)/a_1(2\omega)\}, \qquad (3)$$
$$b_2(\omega)/a_2(\omega) = S_{21}\{a_1(2\omega)/a_2(\omega)\} + S_{22} \quad . \qquad (4)$$

For fixed $|a_1|$ and $|a_2|$, variation of $\angle a_1 - \angle a_2$ generates circles in the complex planes of $b_1/a_1$ and $b_2/a_2$, see Fig.2. The centres of these circles correspond to $S_{11}$ and $S_{22}$ respectively. NA measurements permit $b_1/a_1$ and $b_2/a_2$ to be found by varying $\angle a_1 - \angle a_2$ with a phase-shifter.

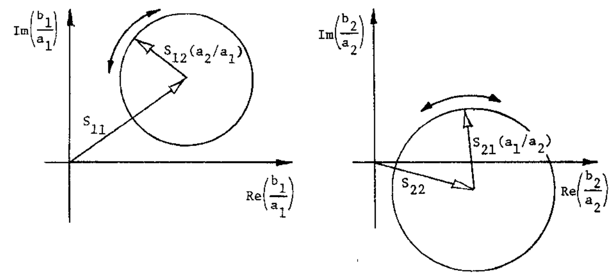


Fig. 2 Circular loci in: (a) $b_1/a_1$-plane at $2\omega$, (b) $b_2/a_2$-plane at $\omega$.

## Measurement Procedure

For each input frequency, three steps are taken:

(i) Reflection coefficients $\Gamma_{mj}$ ($j = 1,2,3$) are measured at reference planes WW' and ZZ' (see Fig.1) as described below.

Fig.3 shows a set-up which applies $a_1(2\omega)$ to the halver input and $a_2(\omega)$ to its output. Alternator $A_1$ sets $|a_1|$, while $A_2$ and P set $|a_2|$ and $\angle a_1 - \angle a_2$ respectively. For accuracy $|a_2|^2$ should be such that $|b_2|^2 - |a_2|^2$ approximates the normal $P_{out}$ to a passive load.

| | Measured value | True value |
|---|---|---|
| Matched load $Z_o$ | $\Gamma_{m1}$ | $\Gamma_{t1} = 0$ |
| Open circuit | $\Gamma_{m2}$ | $\Gamma_{t2} = 1$ |
| Short circuit | $\Gamma_{m3}$ | $\Gamma_{t3} = -1$ |

Hence complex calibration coefficients K,L,M are determined satisfying the bilinear transformation [6]

$$\Gamma_{mj} = (K\Gamma_{tj} + L)/(M\Gamma_{tj} + 1) \ . \qquad (5)$$

Thus for the measured $\Gamma_{mi}$ ($i = 1,2$), here representing $b_i/a_i$, the true values are given by

$$\Gamma_{ti} = (L - \Gamma_{mi})/(M\Gamma_{mi} - K) \ . \qquad (6)$$

(ii) For both $b_1/a_1$ and $b_2/a_2$ measurements, $\Gamma_{mk}$ ($k = 1,2,3$) are found for three settings of $\angle a_1 - \angle a_2$. Extraction of the true values $\Gamma_{tk}$, according to (6), locates three points on one of the circles of Fig.2, the centres of which correspond to $S_{11}$ and $S_{22}$. The external halver impedances are then

$$Z_{in}^{ext} = Z_o(1+S_{11})/(1-S_{11}), Z_{out}^{ext} = Z_o(1+S_{22})/(1-S_{22}). \ (7)$$

(iii)The varactor embedding being known,reflection coefficients $\Gamma_{in}$ and $\Gamma_{out}$ at planes XX' and YY' (Fig.1) can be found using a COMPACT[4] de-embedding file. The large-signal "pumped" varactor impedances $Z_{in}$, $Z_{out}$ are then obtained from $\Gamma_{in}$ and $\Gamma_{out}$.

## RESULTS

The external impedances $Z_{in}^{ext}$ and $Z_{out}^{ext}$ of an experimental halver were measured for $P_{in}$= 16.4dBm and $|b_2|^2 - |a_2|^2$= -3 dBm, the input frequency being

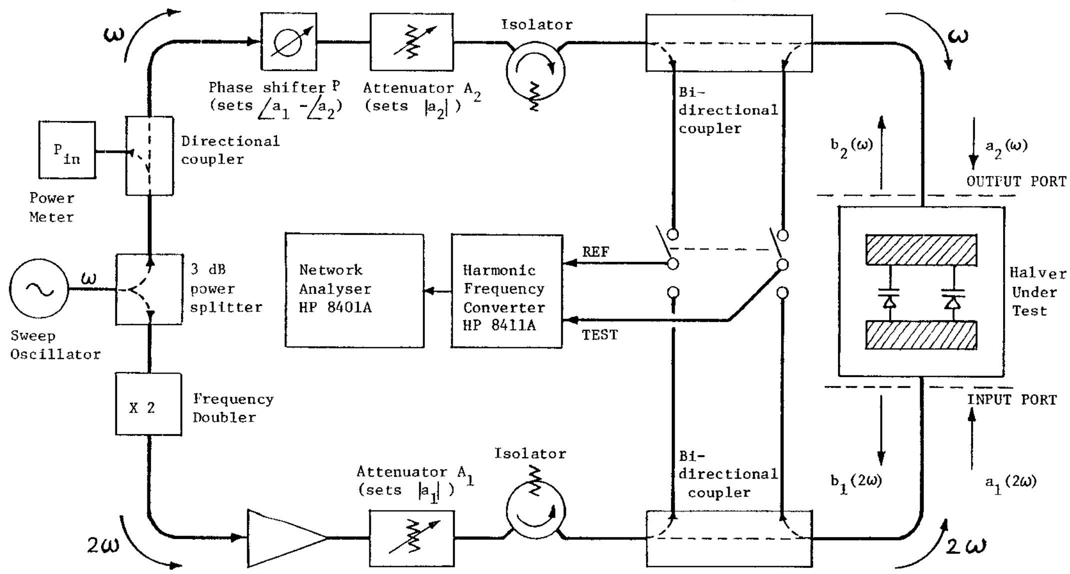


Fig. 3 Two-frequency test set-up.

varied over the 3.5 - 7.0 GHz octave. Fig.5 compares the de-embedded interior impedances $Z_{in}$ and $Z_{out}$ with theoretical estimates given by the Penfield and Rafuse equations[3] modified to account parasitics. For two varactors in parallel at $2\omega$:

$$\tilde{Z}_{in}(2\omega) = \tfrac{1}{2}\left[r_s + \frac{m_1^2}{2m_2}\cdot\frac{(1+V_V/\phi)^{\gamma}}{2\omega C_j(0)}\right] - j\tfrac{1}{2}\left[\frac{1}{2\omega C_j(0)} - 2\omega L_s\right],$$

and for two varactors in series at $\omega$: (8)

$$\tilde{Z}_{out}(\omega) = 2\left[m_2\frac{(1+V_B/\phi)^{\gamma}}{\omega C_j(0)} - r_s\right] - j2\left[\frac{1}{\omega C_j(0)} - \omega L_s\right]. \quad (9)$$

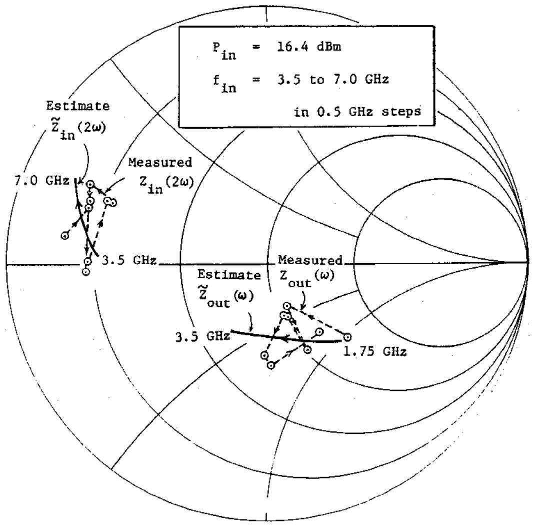


Fig. 5 Comparison between measured $Z_{in}(2\omega)$ and $Z_{out}(\omega)$ of the "pumped" varactors and the estimates $\tilde{Z}_{in}(2\omega)$ and $\tilde{Z}_{out}(\omega)$. Varactors: Frequency Sources, GHZ Division, type GC-1504.

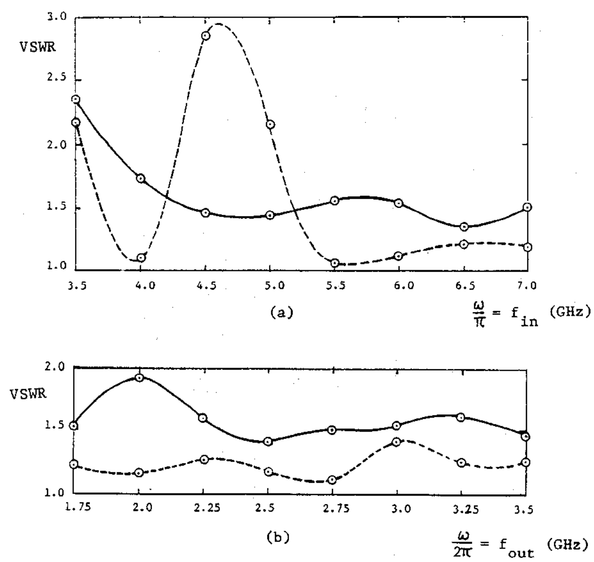


Fig.6. Measured (–) and predicted (---) VSWRs (a) at the input plane WW′; (b) at the output plane ZZ′.

Here $C_j(0)$ is the junction capacitance at zero bias, $\phi$ is the contact potential and $\gamma$ is the capacitance law exponent. The varactor series resistance and inductance are $r_s$ and $L_s$ respectively. The modulation ratios at the input and output frequencies are taken as $m_1 \simeq 0.2$, $m_2 \simeq 0.1$, corresponding to maximum $P_{out}$ conditions [3].

Fig. 5 shows satisfactory agreement between the measured varactor impedances $Z_{in}(2\omega)$, $Z_{out}(\omega)$ and the estimates $\tilde{Z}_{in}(2\omega)$, $\tilde{Z}_{out}(\omega)$. In this case a single iteration suffices.

Fig. 6 compares the measured and predicted external VSWRs of the complete halver. The discrepancy in Fig. 6(a) is caused by input circuit tuning adjustments not included in the calsulation. The measured and calculated output VSWRs, however, are in good agreement, see Fig. 6(b), since the input circuit was not altered in any way.

## CONCLUSIONS

It has been demonstrated that a "two-frequency" variant of Mazumder's method provides an effective and accurate approach to the determination of the "pumped" input and output impedances of a frequency halver, which are required for design centering. A similar approach could be used for doubler design; in that case a halver would replace the doubler in the test set-up.

## ACKNOWLEDGEMENT


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## REFERENCES


[1] R.G. Harrison, "A broadband frequency divider in waveguide", IEEE MTT-S International Microwave Symposium, Ottawa, June 1978, Digest pp. 257-259.

[2] R.G. Harrison, "Theory of the varactor frequency halver", IEEE MTT-S International Microwave Symposium, Boston, June 1983, Digest pp. 203-205.

[3] P. Penfield and R.P. Rafuse, "Varactor Applications", Cambridge, MA : MIT Press, 1962, pp. 454-456.

[4] "COMPACT User's Manual", Comsat General Integrated Systems, Palo Alto, CA, 1979.

[5] S.R. Mazumder, P.D. van der Puije, "Generation of load-pull contours of microwave transistors by electrical simulation of the load impedance", International J. Electronics, Vol. 46, No. 2, 1979, pp. 217-221.

[6] B.A. Syrett, "Computer-aided large-signal measurement of IMPATT-diode electronic admittance", IEEE Trans. Microwave Theory and Tech., Vol. MTT-27, No. 10, Oct. 1979, pp. 830-834.